Predictive Control Barrier Functions for Online Safety Critical Control

61st IEEE Conference on Decision and Control
Cancún, Quintana Roo, Mexico, December 6th 2022

Joseph Breeden, Dimitra Panagou

Department of Aerospace Engineering
University of Michigan, Ann Arbor, MI, USA

UNIVERSITY OF MICHIGAN

Introduction - Control Barrier Functions

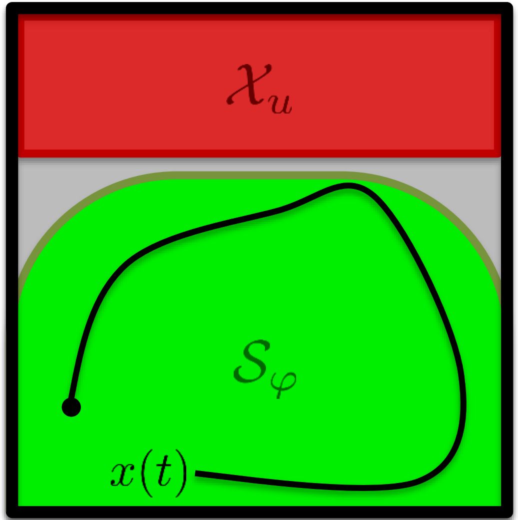
- Control Barrier Functions (CBFs) [1] are a tool for set invariance
  - Let $\mathcal{T} \subseteq \mathbb{R}$ be a time-domain and $\mathcal{X} \subseteq \mathbb{R}^n$ be a state domain
  - Control-affine system: $\dot{x} = f(t,x) + g(t,x)u$
  - Let $\mathcal{X}_u$ denote the set of unsafe states (e.g. states that correspond to collisions with obstacles)
  - Given a function $\varphi : \mathcal{T} \times \mathcal{X} \to \mathbb{R}$ and class-$\mathcal{K}$ function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, the condition [1]

$$\dot{\varphi}(t,x,u) \leq \alpha(-\varphi(t,x))$$

    is sufficient to render the state trajectory $x(t)$ always inside

$$\mathcal{S}_\varphi(t) \triangleq \{x \in \mathcal{X} \mid \varphi(t,x) \leq 0\}$$

  - Design $\varphi(t,x)$ so that $\mathcal{S}_\varphi \cap \mathcal{X}_u$ is empty

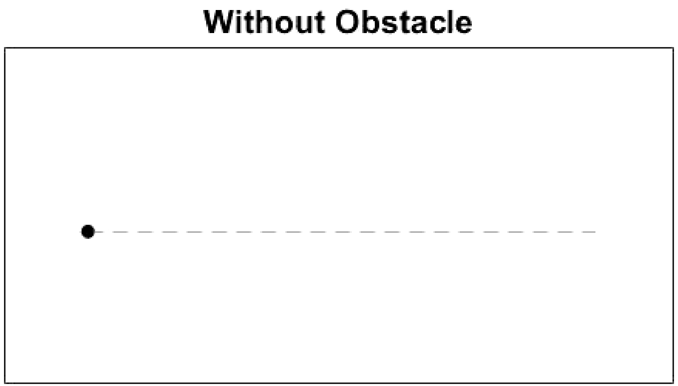


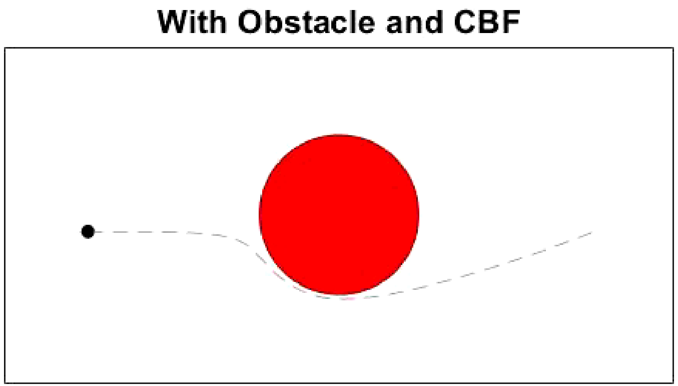
[1] Ames et al, “Control barrier functions: Theory and applications”, ECC 2019

Introduction – Online Safety-Critical Control

- CBFs are commonly implemented via online modifications of a nominal control law using the quadratic program

$$u = \arg\min_{u \in \mathbb{R}^m} \|u - u_{\text{nom}}(t, x)\|^2$$
$$\text{such that } \dot{\varphi}(t, x, u) \leq \alpha(-\varphi(t, x))$$





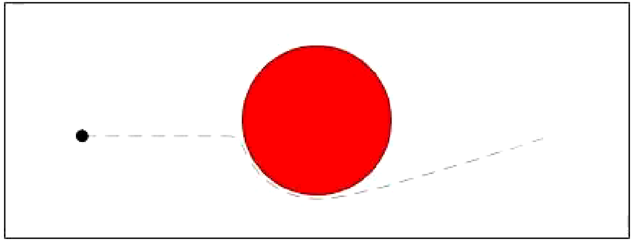
Introduction

- These online modifications often result in
  - Aggressive control responses

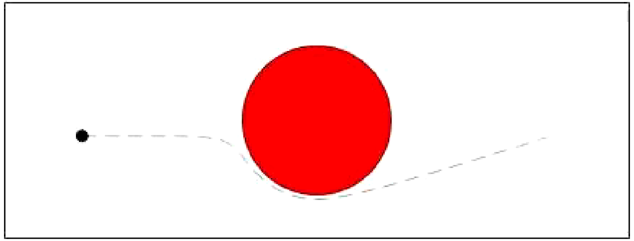
    and/or

  - Inefficient/late control responses

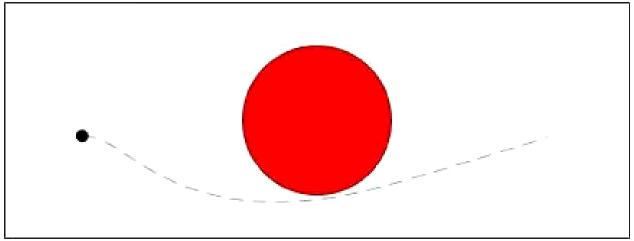
Hard Stop



Input Constrained



With Obstacle Predictions

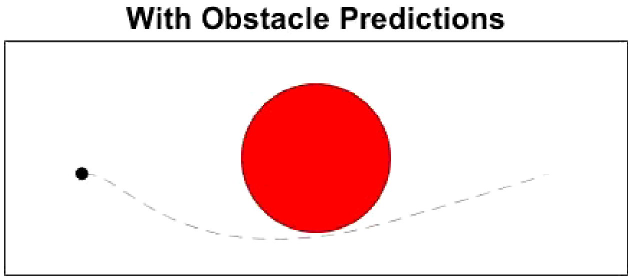


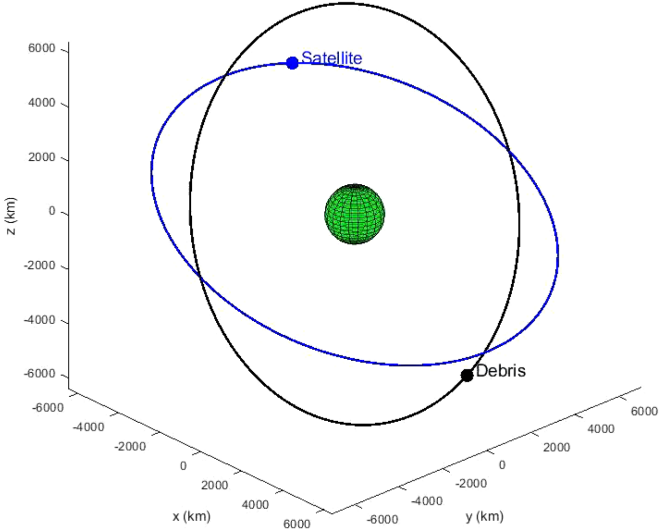
- Hypothesis: Considering the future trajectories of the system when choosing the present control input will mitigate the above behaviors
  - This is the guiding principle of Model Predictive Control (MPC) (e.g. [1])

[1] Grandia et al., "Multilayered safety for legged robots via control barrier functions and model predictive control", ICRA 2021

Problem Statement

- Develop a CBF that is aware of the future trajectory of the system under u_{nom} on a finite horizon $[t, t+T]$ and that adjusts this trajectory long before safety is violated





Overview

1. Defining the "future trajectory" of the system
2. Analyzing the future trajectory
3. Encoding the "Predictive CBF"
4. Simulations
5. Discussion

Preliminaries

- System: $\dot{x} = f(t,x) + g(t,x)u$
- Control input unconstrained, i.e. $u \in \mathcal{U} = \mathbb{R}^m$
- Safety function $h : \mathcal{T} \times \mathcal{X} \to \mathbb{R}$ and safe set

$$\mathcal{S}_h(t) = \{x \in \mathcal{X} \mid h(t,x) \le 0\}, \;\; \mathcal{S}_h \cap \mathcal{X}_u = \emptyset$$

where h is not a CBF, and can be of any relative-degree

Definition. An absolutely continuous function $\varphi : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a Control Barrier Function (CBF) for the set $\mathcal{S}_\varphi$ if there exists $\alpha \in \mathcal{K}$ such that

$$\inf_{u \in \mathbb{R}^m} \Big[\underbrace{\partial_t \varphi(t,x) + L_{f(t,x)+g(t,x)u}\varphi(t,x)}_{=\frac{d}{dt}[\varphi(t,x)]} \Big] \le \alpha(-\varphi(t,x))$$

for almost every $x \in \mathcal{S}_\varphi(t), t \in \mathcal{T}$, where $\mathcal{S}_\varphi(t) \triangleq \{x \in \mathcal{X} \mid \varphi(t,x) \le 0\}$.

Defining the "Future Trajectory"

- Suppose a nominal control input $u_{\mathrm{nom}} : \mathcal{T} \times \mathcal{X} \to \mathbb{R}^m$

Definition. The function $p : \mathcal{T} \times \mathcal{T} \times \mathcal{X} \to \mathcal{X}$, denoted $p(\tau; t, x)$, satisfying $p(t; t, x) = x$ and

$$\frac{\partial}{\partial \tau} p(\tau; \cdot) = f(\tau, p(\tau; \cdot)) + g(\tau, p(\tau; t, x)) u_{\mathrm{nom}}(t, p(\tau; \cdot))$$

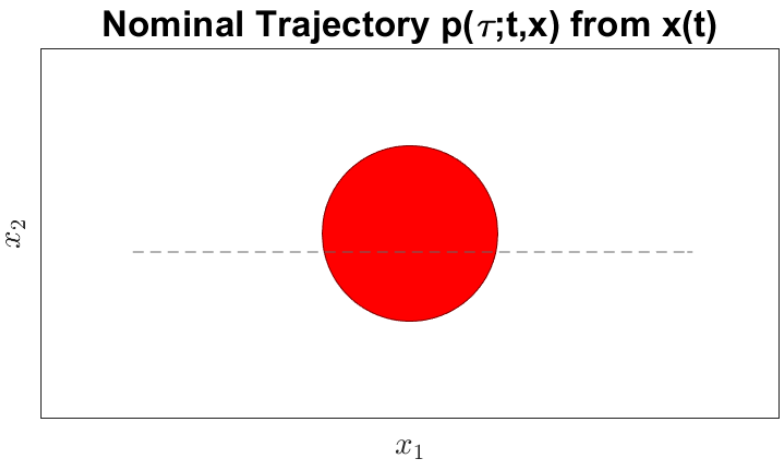
for all $\tau \geq t$ is called a *path function.*

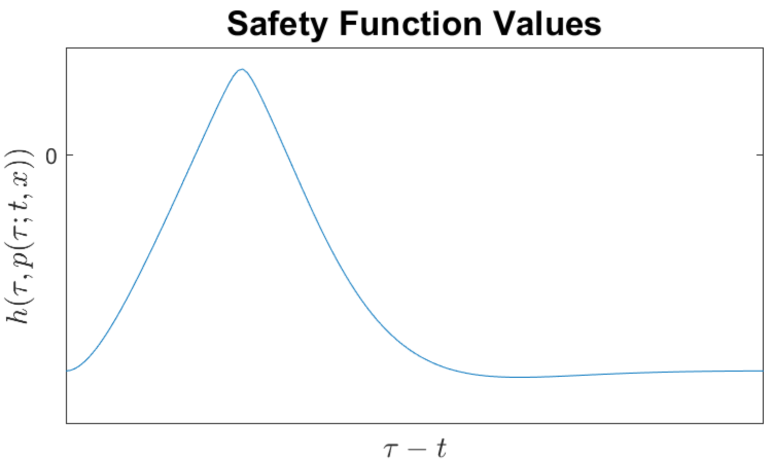
- p is potentially unsafe, so this is not a "Backup CBF" [1-3]

[1] Squires et al., "Constructive barrier certificates with applications to fixed-wing aircraft collision avoidance," CCTA 2018
[2] Chen et al., "Guaranteed obstacle avoidance for multi-robot operations with limited actuation: A control barrier function approach," LCSS 2021.
[3] Wabersich and Zeilinger, "Predictive control barrier functions: Enhanced safety mechanisms for learning-based control," TAC 2022.

Analyzing the Future Trajectory

- Propagate trajectory for receding time horizon $T > 0$
- Compute safety function along the hypothetical trajectory

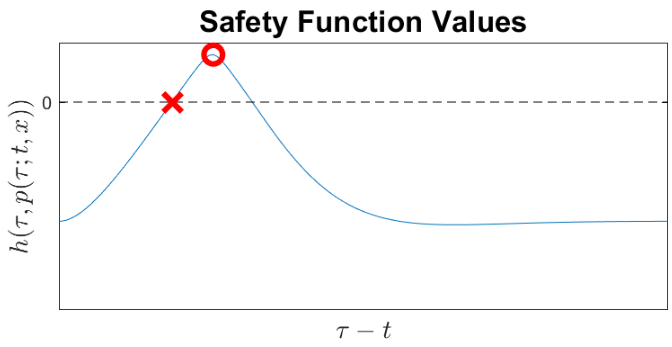




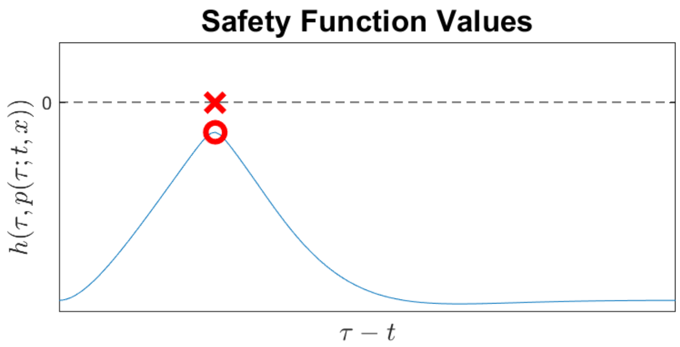
Analyzing the Future Trajectory

- Question: Is the future trajectory (on a finite horizon) safe?

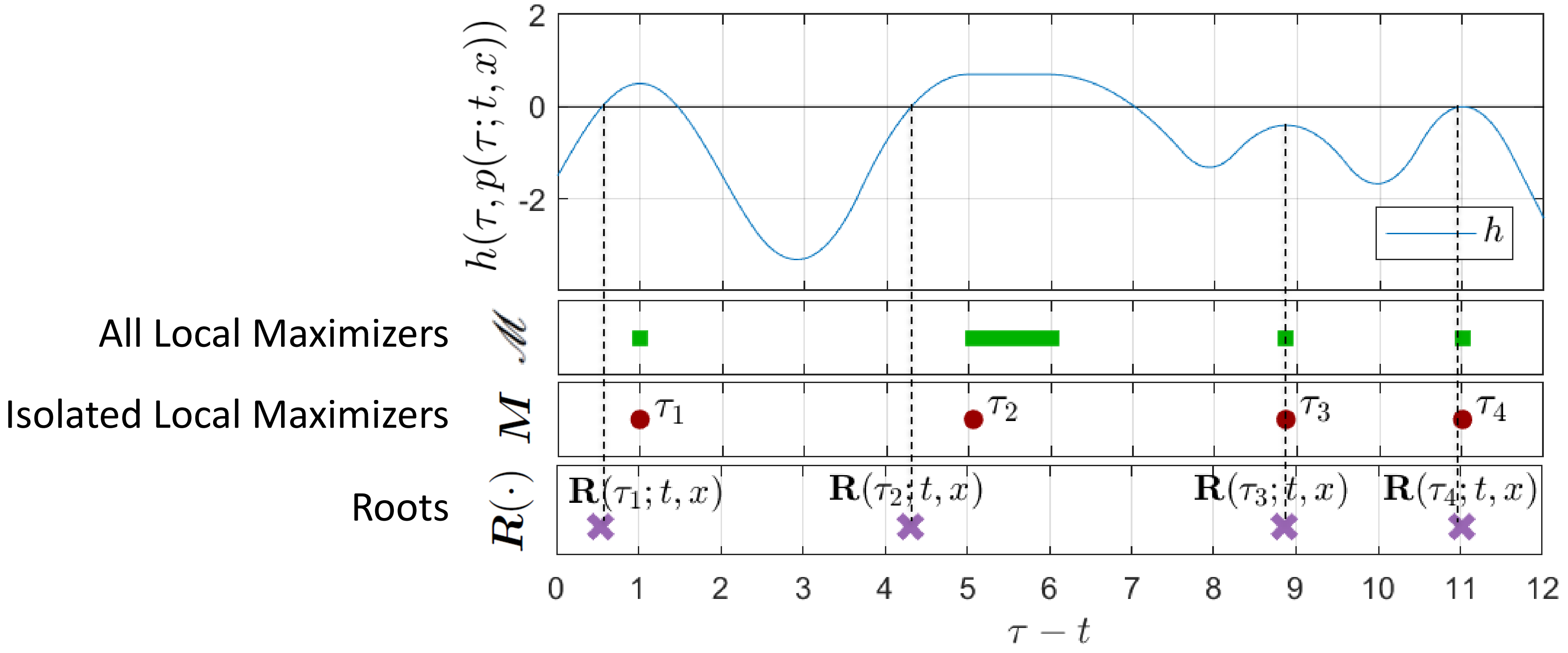
- "No":
  1. When does the trajectory become unsafe?
  2. By how much does the trajectory become unsafe?



- "Yes":
  1. When does the trajectory become least safe?
  2. By how much margin is the trajectory safe?



Times of Interest



Encoding the Predictive CBF

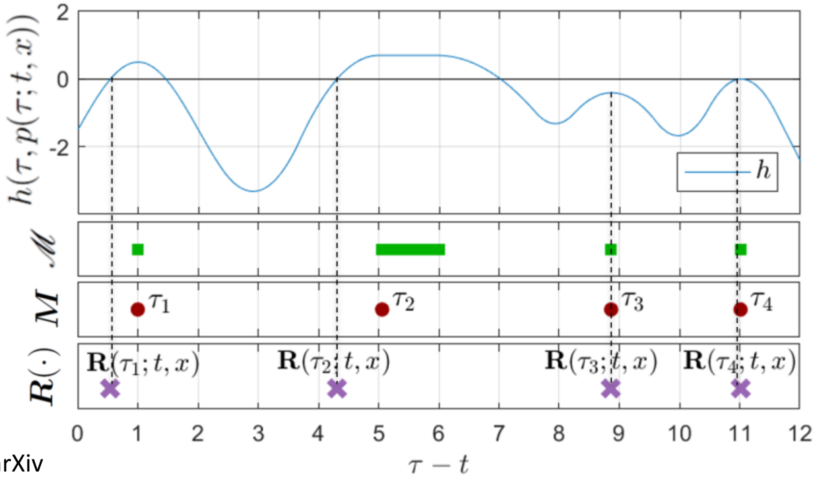
- Given a time $\tau_i \in \boldsymbol{M}(t,x)$ and a nondecreasing function $m_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ define the "Predictive CBFs":

$$H_i(t,x) \triangleq h(\tau_i, p(\tau_i; t, x)) - m_i(\boldsymbol{R}(\tau_i; t, x) - t)$$

Amount by which safety is violated, or amount of margin

Time to make correction*

- Choose m_i so that $H_i(t_0, x_0) \leq 0$



*See also Black et al., "Future-Focused Control Barrier Functions for Autonomous Vehicle Control", arXiv

Main Result

- See next slide for assumptions

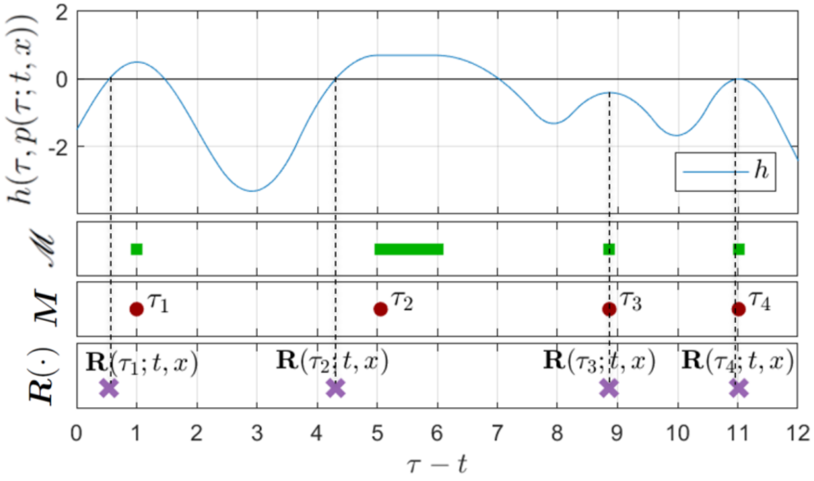
Theorem. Each H_i is a CBF for $\mathcal{S}_{H_i}$, and $\mathcal{S}_{H_1}(t) \subseteq \mathcal{S}_h(t)$ for all $t \in \mathcal{T}$.

$$u = \arg\min_{u \in \mathbb{R}^m} \|u - u_{\text{nom}}(t,x)\|^2$$

$$\text{such that } \underbrace{\partial_t H_1(t,x) + L_f H_1(t,x) + L_g H_1(t,x) u}_{=\dot{H}_1(t,x,u)} \leq \alpha(-H_1(t,x))$$

$$H_i(t,x) = h(\tau_i, p(\tau_i; t, x)) - m_i(\boldsymbol{R}(\tau_i; t, x) - t)$$

$$\mathcal{S}_{H_i}(t) = \{x \in \mathcal{X} \mid H_i(t,x) \leq 0\}$$



Main Result - Assumptions

Boundedness Assumptions:

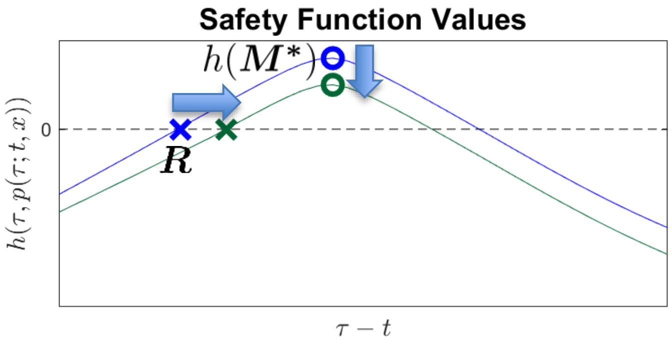
1. $h(t,x)$ is upper bounded by $h_{\max} < \infty$
2. $\frac{d}{d\tau}[h(\tau, p(\tau; t, x))$ is upper bounded by $\gamma < \infty$

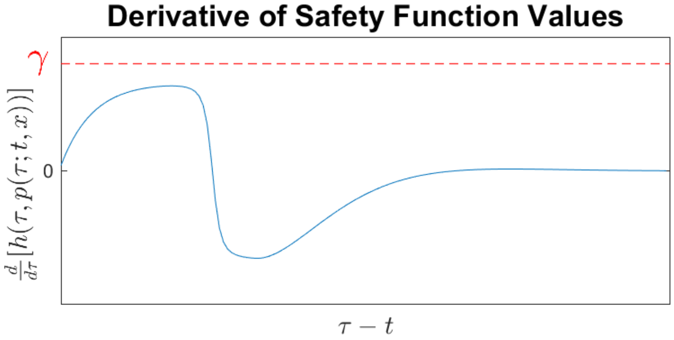
Controllability Assumptions:

3. H_i is absolutely continuous
4. $m_i'(\lambda) > 0$ for $\lambda \in (0, T)$
5. The sensitivity $\frac{\partial h(\eta, p(\eta;t,x))}{\partial x} \frac{\partial p(\eta;t,x)}{\partial x} g(t,x)$ is nonzero when η is not a) t, b) $t+T$, or c) a local maximizer (i.e. in $\mathscr{M}$)

Consistency Assumption:

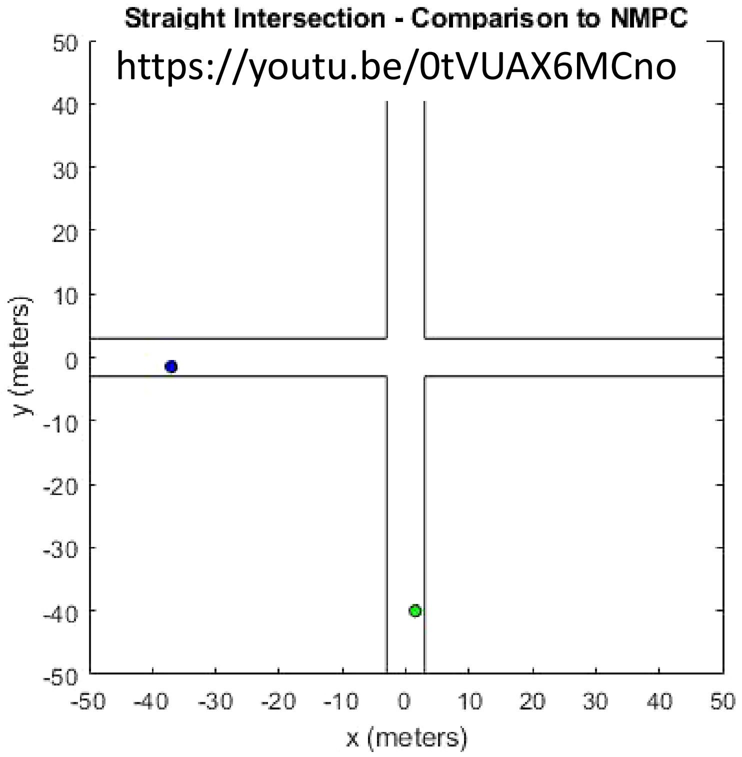
6. $\frac{\partial h(\tau, p(\tau;t,x))}{\partial x} \cdot \frac{\partial h(\eta, p(\eta;t,x))}{\partial x} \geq 0$ when $\eta = \boldsymbol{R}(\tau; t, x)$



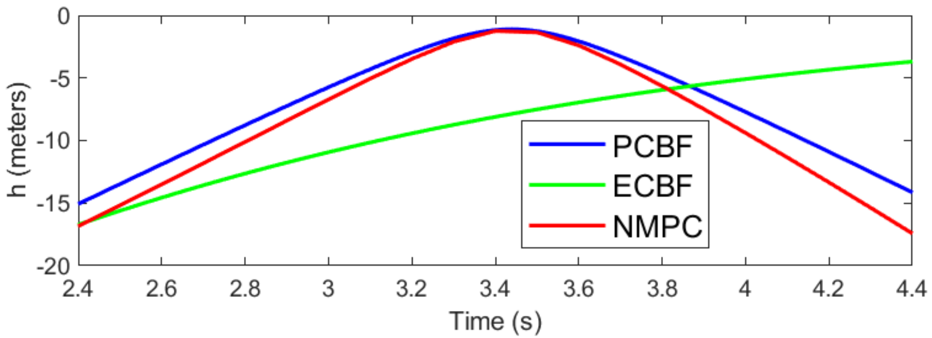


Theorem. Each H_i is a CBF for $\mathcal{S}_{H_i}$, and $\mathcal{S}_{H_1}(t) \subseteq \mathcal{S}_h(t)$ for all $t \in \mathcal{T}$.

Simulation Results – Cars Straight Intersection



ECBF Comparison: Nguyen and Sreenath, "Exponential control barrier functions for enforcing high relative-degree safety-critical constraints", ACC 2016



- Safety requirement

$$h = \rho - \|l_1(z_1) - l_2(z_2)\|$$

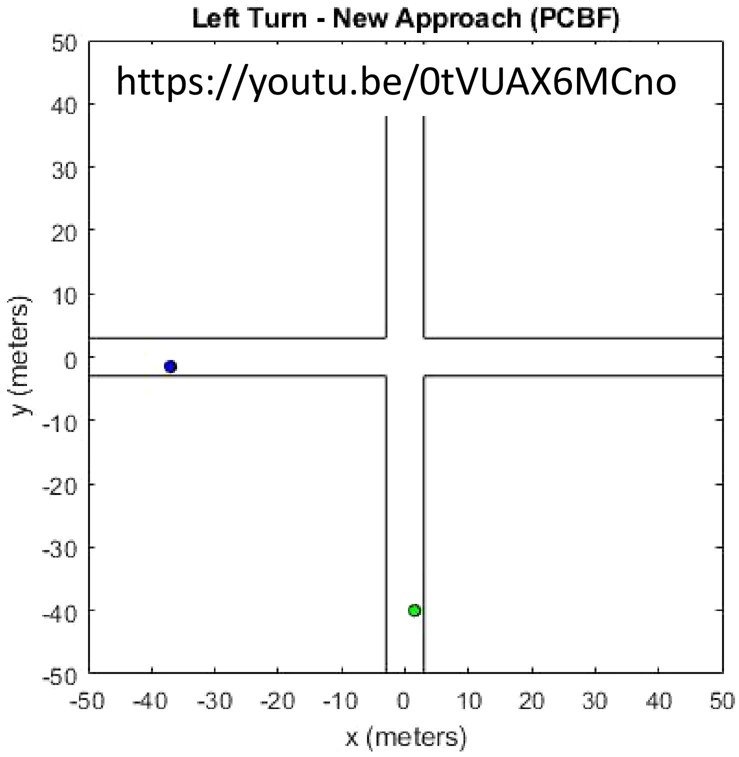
- Safe control input is

$$u = \arg\min_{u \in \mathbb{R}^2} \|u - k([\dot{z}_1,\ \dot{z}_2]^{\mathrm{T}} - v_{\mathrm{cmd}})\|^2$$

$$\text{such that } \dot{\varphi}(t, x, u) \leq \alpha(-\varphi(t, x))$$

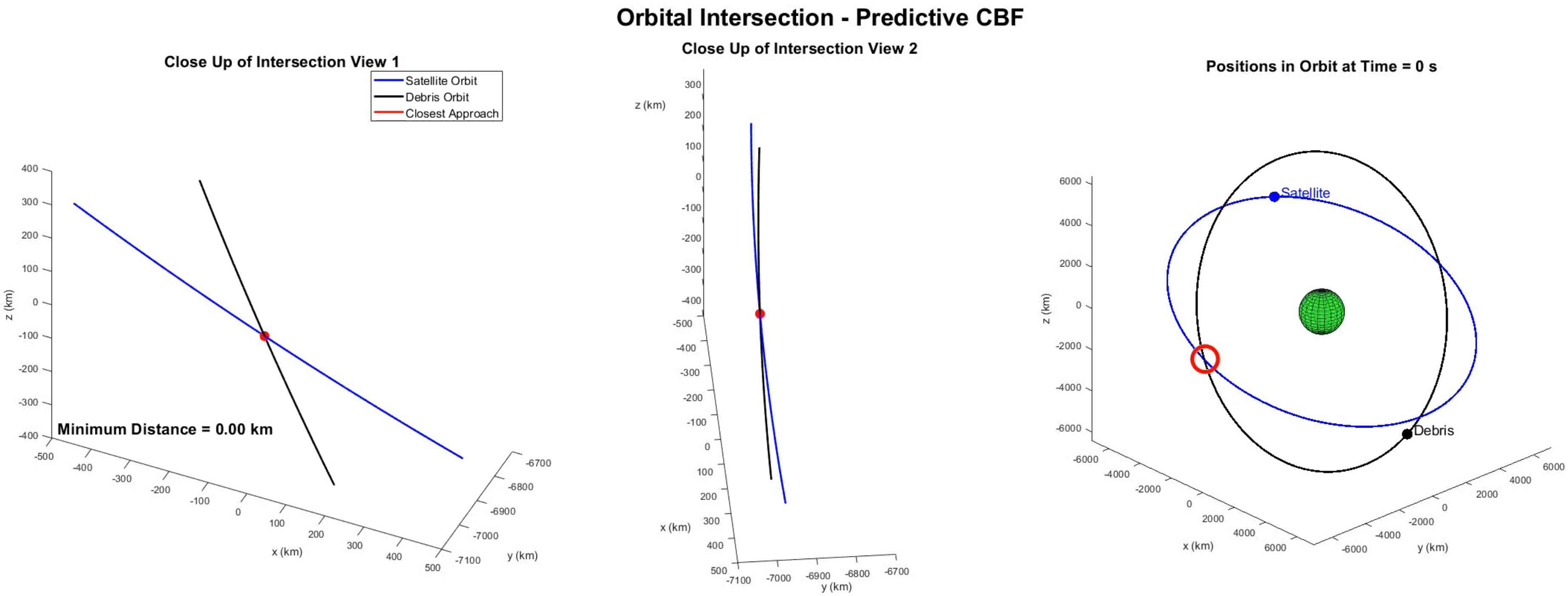
where $\varphi \in \{H_{\mathrm{ecbf}}, H_1\}$.

Simulation Results – Cars Left Turn Intersection



- Average control computation times:
  - ECBF: 0.0011 s
  - PCBF: 0.0061 s
  - NMPC: 0.40 s
- Simulations in MATLAB
- ECBF + PCBF controller computed with quadprog
- NMPC controller computed with nlmpc + fmincon using SQP algorithm limited to 8 iterations

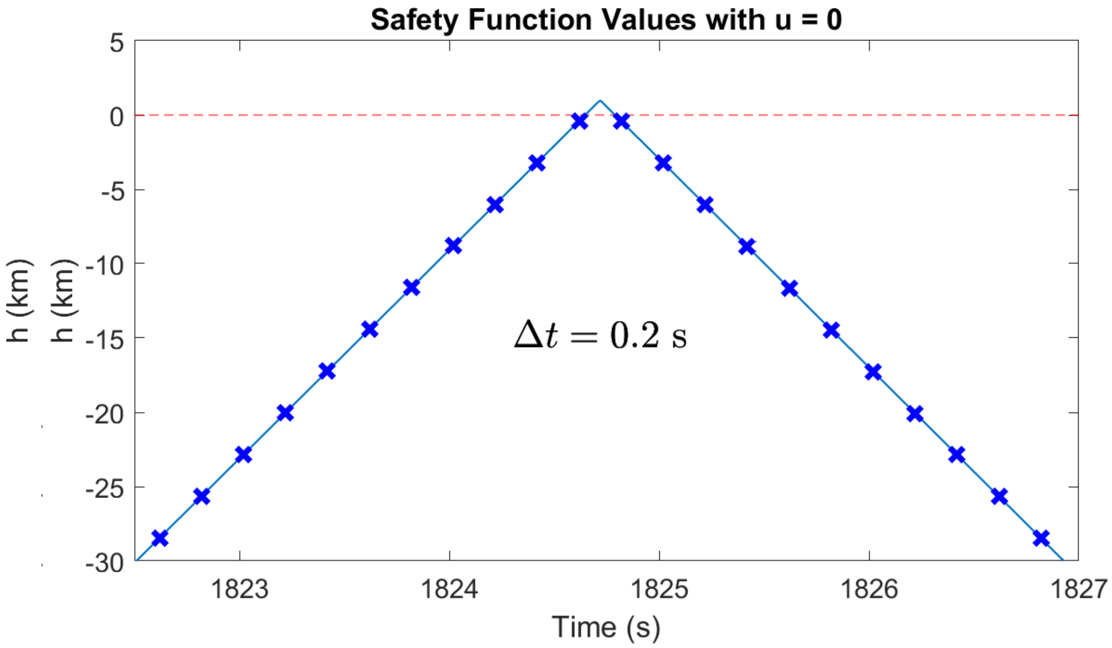
Simulation Results - Satellites



https://youtu.be/HhtWUG63BWY

Predictive CBF thrusts a quarter orbit in advance when less control effort is required.

Simulation Results - Satellites



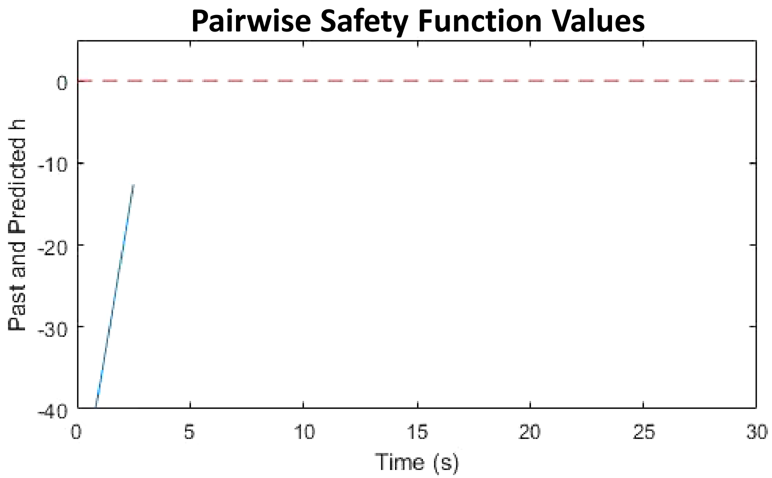
- Satellites have a 1 km radius keep out zone
- Satellites orbit at 7.5 km/s
- The minimum sample time to guarantee detection of an unsafe state is 0.143 s
- At this discretization interval, NMPC with the same horizon as the PCBF would require 9800 samples, which is impractical.

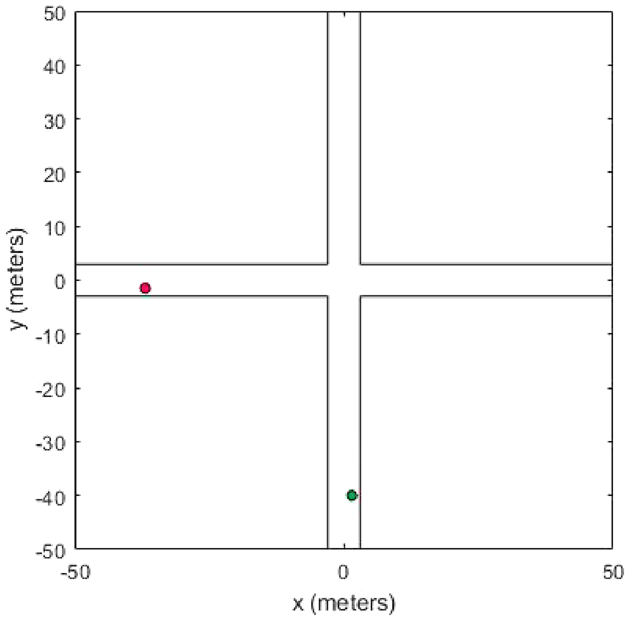
Conclusions

- We have presented a new framework for constructing CBFs for generic safety functions h using future trajectory predictions
- The Predictive CBF H_1 takes into account the future trajectories the system is expected to follow and modifies these trajectories before reaching unsafe states
- Compared to MPC, the Predictive CBF
  - followed similar trajectories in simulation
  - yields a pointwise control-affine safety constraint
    - Results in a convex QP control law even for nonlinear dynamics and constraints
    - QP is m-dimensional (where $u \in \mathbb{R}^m$) instead of mN-dimensional as in MPC
  - evaluates safety over a continuous predicted trajectory without fixed sampling (important for satellite simulations or other rapidly evolving systems)

Ongoing Work

- Provably guaranteed input constraint satisfaction
  - Currently, input constraint satisfaction is achieved via tuning
- Distributed Systems
- Predictions with uncertain obstacles
- Improving a specified cost metric (similar to MPC)





Thank You To Our Sponsors

Predictive Control Barrier Functions for Online Safety Critical Control

61st IEEE Conference on Decision and Control
Cancún, Quintana Roo, Mexico, December 6th 2022

Joseph Breeden, Dimitra Panagou

Department of Aerospace Engineering
University of Michigan, Ann Arbor, MI, USA

UNIVERSITY OF MICHIGAN